MATH 147 QUIZ 6 SOLUTIONS

1. For the iterated integral $\int_0^1 \int_x^1 xy\, dy\, dx$, interchange the order of integration and evaluate. (5 points)

   We note that the boundary $0 \leq x \leq 1$, $x \leq y \leq 1$ is equivalent to $0 \leq y \leq 1$, $0 \leq x \leq y$. One could arrive at this conclusion by sketching the triangle by hand, or solving the inequalities. Therefore, we can interchange the order and instead solve the integral

   $$\int_0^1 \int_0^y xy\, dx\, dy.$$

   This evaluates as follows:

   $$\int_0^1 \int_0^y xy\, dx\, dy = \int_0^1 \left[\frac{x^2 y}{2}\right]_0^y dy = \int_0^1 \frac{y^3}{2} dy = y^4/8\big|_0^1 = 1/8.$$

   Note that an inaccurate bound like $y \leq x \leq 1$ gives the right *numerical* answer, but is not correct. The region would be flipped in this case, but the area is preserved due to symmetry in x and y.

2. Convert the double integral $\int\int_D x^2 + y^2\, dA$ into a double integral involving polar coordinates, where D is the region in $\mathbb{R}^2$ bounded by the curve $r = \cos(2\theta)$, with $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$. DO NOT calculate the integral. (5 points)

   We make the polar substitutions $x = r\cos(\theta), y = r\sin(\theta)$, which result in the additional identity $r^2 = x^2 + y^2$. Upon making this substitutions, and changing $dA \to r\, dr\, d\theta$, we get

   $$\int_{-\pi/4}^{\pi/4} \int_0^{\cos 2\theta} r^3\, dr\, d\theta.$$